The therapy to shock therapy: optimal dynamical policies for transition economies

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We show that a simple model reproduces very closely the evolution of the GDP in constant dollars of many countries during the times of recession and recovery. A theoretical analysis illustrates how an optimal dynamical policy reduces both recession duration and severity, and increases the value of GDP at all times. We propose a criterion to distinguish a posteriori a dynamical policy from a static one.

I. INTRODUCTION

Explaining growth and recessions has been central to economics ever since its begining. The recent collapse of the former block of European communist countries gave much to analyze and comment (see [1] for a review). However, since recession and recovery are usually split into distinct periods, the factors of decline and growth have been investigated separatly (e.g. [1, 2, 3]) and little attention has been devoted to the intrisic universal relationship between recession and recovery. We take the position that after a dramatic change in the economy, the very same process responsible for the recovery is already at work in the recession. Instead, rather than being afraid of addressing head-on the recession conditions one make them part of the process of recovery. Thus the focus of the policy planning is transfered from the negative aspect related to blocking the economic collapse to the optimization of the recovery process as a whole.

The capability of the government to steer economic recovery encompasses taxation and redistribution, which is a small part of the processes and phenomena taking place. We show however that in fact when used optimally, according to an understanding of the underlying economic processes, even this very limited set of tools can lead to remarkably better results.

In particular we find that quite universally, significant reforms (even the most ultimately successful ones) are followed by a period of decay. Thus, as an important lesson for policy makers, we derive that the success of a reform is to be evaluated by a more sensitive analysis that we describe below. Such an analysis discriminates between the negative effect that the reforms have on the old "order" and the growth induced by the reforms.

In section 2, to validate our model in well-defined conditions, we consider the evolution of the former communist block economies following their liberalization. We find that our "universal recession-recovery model" fits all of them, and also some other countries.

In section 3, we describe in detail our model. The main idea is that following dramatic events, large sectors previously dominating the economy start fading away while previously undevelopped sectors take over. The recession-recovery process is then completely determined by the value of the returns for the two aggregate sectors and the transfer rate of economic activity between them.

In section 4, we exploit the understanding of section 3 in order to find an optimal dynamic schedule for the transfer rate between the fading and the taking-over sectors. More specifically, we are looking for transfer policy (governed by the taxation-subsidy balance) that minimizes the depth and duration of the recession, and maximises both the GDP value and the final growth rate. We finally propose a means to differentiate static from dynamical policies in historical data.

II. THE DATA

The GDP of many countries shows dramatic and sudden decrease followed by a slow recovery, a pattern commonly known in other areas of Economics as J-shape. Excluding countries affected by external causes such as wars and petroleum shocks, one is left with about 28 countries. As it turns out, most of these recessions were caused by political reforms, taxation or financial crises. Figure 1 reports the evolution of the GDP in constant dollars of several countries, revealing a common pattern. These recessions can be characterised by their intensity (maximum loss of GDP), the time of the minimum GDP and the time to recover the previous level of economic production.

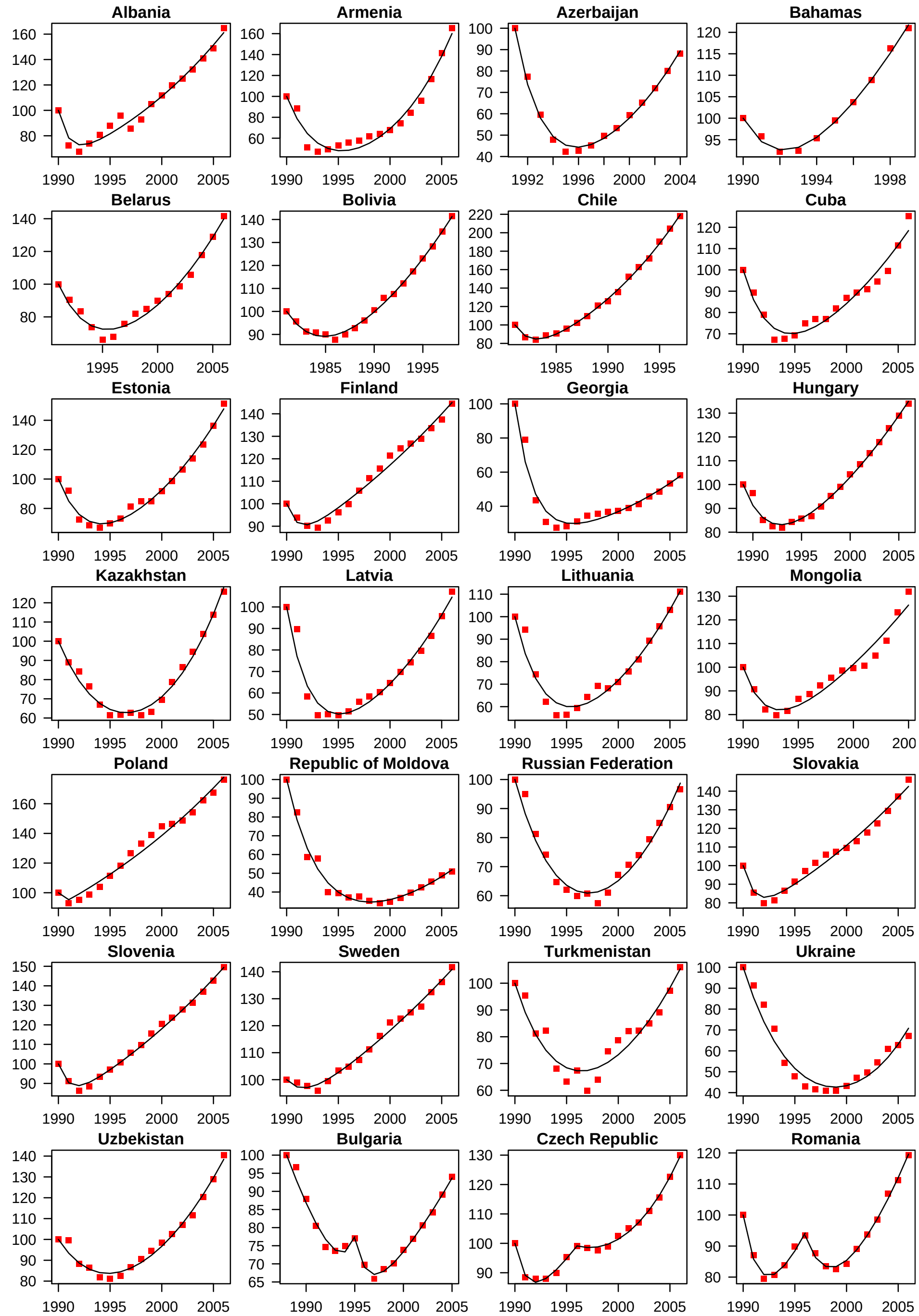

FIG. 1: Time evolution of the GDP in constant dollars of 28 countries, and the fits to Eq. (1)

Remarkably, all previously communist eastern European countries have experienced a lasting recession followed by a recovery. Some countries such as Romania, Bulgaria and the Czech Republic clearly display a double dip (Fig. 1). As it happens, the onset of the second dip of Romania and Bulgaria is unambiguously related to an alternance of power, substantiating our claim that wrong taxation policies, or their implentation, are to be blamed for further degradation of the situation. The severity and duration of the recession varies widely between the countries: Poland has recovered the quickest, while some countries such a Russia, Latvia had just come back to their previous GDP in 2006. The Republic of Moldova was still 50% down in 2006. These differences raise two questions: what is their causes? How efficient was the transition policy?

For reasons explained below, we parametrize the GDP J shapes as displayed in Fig. 1, in terms of the following formula

$$W(t) = W(t_0)[fe^{\lambda^+(t-t_0)} + (1-f)e^{\lambda^-(t-t_0)}], \tag{1}$$

where

- $W(t_0)$ is the initial GDP at the time t_0 of the reform
- f is the fraction of the economy that grows at rate λ^+;
- the rest of the economy $(1-f)$ deflates at rate λ^-

A J-shaped W is obtained if $W'(t_0) < 0$, i.e. if $f\lambda^+ + (1-f)\lambda^- < 0$.

The fit of recession and recovery times of 28 countries is remarkable (the details are reported in appendix A), and may a priori suggest a surprising degree of policy constancy; however, as discussed in section 4, dynamical policies can also be relatively well fitted with the same model and one needs more data than the GDP time series to detect them. Such sudden drops are signatures of shocks. Nevertheless, the fits are not perfect, the recovery part often displaying some irregularities in the recovery phase. most of them negative. We could trace possible causes of some of them to additional shocks: Albania suffered from a bank crisis in 1996, Poland from an "ambitious tax reform" in 2000, and Mongolia from large livestock loss in the same year. The worst secondary shocks were born by Romania and Bulgaria (1996 elections), and Czech Republic (2000 bank crisis) which resulted into a second dip. This raises the question on whether the reforms were successful or on the contrary detrimental: comparing the values of λ^+, λ^- and f before and after the second dip, one concludes that, according to our model, the crisis in Czech Republic had long-term positive effects, both the rates of expansion and decline having much improved, at the cost of the initial fraction of the expanding sector. The case of Romania is best described as *bis repetita (non placuerunt)*: the second crisis leading to almost the same fitting parameters as the first one. Finally, Bulgaria has spurious results regarding the first shock, which is due to the fact that GDP was mostly decreasing. Our fit indicates that the growing part was very small (from 0.2% to 2%), but doing very well, and an immense part steadily decreasing. This is clearly wrong and easily reproduceable with other shocks if one restricts the data so as to include a very small part of the recovery. The figures obtained for the second shock are in line with all the other shocks.

III. THEORETICAL MODEL

The rationale behind the fitting function used in the previous section is the following. The after-shock economy is supposed to consist in two sectors [7], one with activity w_1, growing intrisically at rate $\alpha_1 > 1$, and the other one with activity w_2 but instrisically shrinking ($\alpha_2 < 1$). They interact through economic activity transfer taking place at rate β, according to the difference of activity. Mathematically,

$$\frac{\partial w_1(t)}{\partial t} = \alpha_1 w_1(t) + \beta[\langle w\rangle(t) - w_1(t)] \tag{2}$$

$$\frac{\partial w_2(t)}{\partial t} = \alpha_2 w_2(t) + \beta[\langle w(t)\rangle - w_2(t)] \tag{3}$$

The governement takes a fraction $\beta/2$ of the difference of activity between the two sectors from the largest sector and gives it to the smallest one. This means in particular that, when the instrisically expanding sector represents a small part of the economy, the government subsidises it by transfering resources from the shrinking sector, thereby accelerating the transition. Note that because of redistribution, both sectors end up growing at the same rate. Therefore, it is wrong to think of the dynamics of this model as describing a growing sector and a declining sector since both have a growing and a declining part.

Solving the dynamics of this system is straightforward by computing the eigenvalues and associated eigenvectors. The two eigenvalues are

$$\lambda^\pm = \frac{\delta[\sigma/\delta - \zeta \pm \sqrt{1+\zeta^2}]}{2} \tag{4}$$

where $\delta = \alpha_1 - \alpha_2$, $\sigma = \alpha_1 + \alpha_2$ and $\zeta = \beta/\delta$. These eigenvalues correspond to the rates measured in the previous section. The unnormalized eigenvectors are $(\zeta, -1 \pm \sqrt{1+\zeta^2})$. Let us denote by $\mathbf{v}^\pm = (v_1^\pm, v_2^\pm)$ the respective orthonormal eigenvectors. Following standard procedure, one decomposes $\mathbf{w}(t=0)$ into the basis $\mathbf{v}^\pm$, obtaining $\mathbf{w}(t) = \omega^+\mathbf{v}^+e^{\lambda^+ t} + \omega^-\mathbf{v}^-e^{\lambda^- t}$ where $\omega^\pm = \mathbf{w(0)}.\mathbf{v}^\pm$ are the projections of the initial conditions onto the sector decomposition described above. In other words, both w_1 and w_2 have an increasing and a decreasing part. The steady state is reached when the importance of the negative component is vanishingly small compared two the positive component both for w_1 and w_2. The typical time for reaching this asymptotical regime is $O(\lambda^+/|\lambda^-|)$ units of time. Then the two groups grow at the same rate, λ^+ (Fig. 5). In this regime, the growth of the negative component is entirely due to the transfer of activity from the positive one[8].

We shall be interested in this paper in the total economic activity $W = w_1 + w_2$ and shall consider the GDP as its proxy. Also we are interested in the dynamics of inequality between the sectors, measured by $\Delta = w_1/w_2$. Note that the empirical data determine only partially the parameters of formula (1)of previous section: while the rates $\lambda^\pm$ can be measured directly, more detailed information is needed in order to determine all three parameters α_1, α_2 and β. This is due to the fact that f does not correspond directly to $w_1(0)$ since even at the beginning sector 2 has a growing part (i.e. $v_2^+ \neq 0$).

A. Static policy making

Assume that the rate α_1 and α_2 are constant and fixed by constraints beyond the control of the government. The government's only influence is in the transfer rate through the tax rate policy. This in itself is a very powerful instrument that the government is pressed to use: indeed economic activity is linked to employment, and a fast-shrinking sector implies growing social inequality and voters dissatisfaction. If the rate of shrinking is much faster than the rate of labor transfer between the two sectors, inequality at the sector level translates into growing social inequality. In that sense, the inequality between sectors is an upper bound to social inequality. It should be noted that β is the effective rate of transfer, not the one hoped for by the government; indeed, if the latter is not able to collect taxes or if its authority is undermined by inadequate rule of law due to the collapse of institutions, the effective β may turn out much smaller.

The final growth rate depends much on the policy: increasing β reduces both eigenvalues, hence the total growth rate *in the steady state*: maximal asymptotic economic growth is achieved when there are no taxes. This seems to substiante the claims of the so-called supply-side economics (see e.g. [4]). However, global growth rate is not the only success measure of a taxation policy: inequality is also to be taken into account.

Indeed, since $\lambda_- < 0$, group 2 would simply disappear in the absence of redistribution. Decision makers who only focus on growth will therefore take β as small as socially responsible and electorally possible. Some others will try to minimize inequality. Since both w_1 and w_2 end up growing at the same speed, their asymptotic ratio $\Delta = \lim_{t\to\infty} w_1(t)/w_2(t)$ is a measure of economic inequality. Using basic algebra, one finds that

$$\Delta = v_1^+/v_2^+ = 1/(-1/\zeta + \sqrt{1+\zeta^2}) \simeq 2/\zeta = 2(\alpha_1 - \alpha_2)/\beta \tag{5}$$

if $\zeta \ll 1$, in which case reducing the inequality by a half requires to double the transfer rate; in addition, inequality is proportional to the difference of growth rate.

Since the rates are fixed by assumption, inequality only depends on policy, not on initial conditions. Inequality ceases to exist only for large β, which is ideal communism, at the cost of growth rate.

Therefore, a head of state of a country about to convert from communism to capitalism can choose between a small but long recession with anemic final growth, or a large but short-lasting recession with large final growth (Fig. 2). A cynical politican would ensure that the wealth of the majority of voters has increased by the end of his tenure or at least that the recovery has begun. In addition, he can claim no personal responsibility in the resulting social inequality.

IV. GRADUALISM VERSUS SHOCK THERAPY

As shown in section 1, all the Eastern European countries have experienced economic recession when switching from communism to capitalism. The variety of intrisic growth and decline rates and policies yielded vast differences between speed of recovery and depth of recession of various countries. Understandably a large corpus of literature investigates what factors could explain this variety of behaviours (e.g. [1, 3]). In particular, the technique of making the transition abrupt and short has been labelled as "shock therapy" [2]. The concept of shock therapy has been the focus on long debates which have not been settled to this very day [4]. The other approach is called gradualism, and advocates to follow a more gentle rythm [2].

Our model makes it possible to investigate this issue. We shall assume that α_1 and α_2 are intrisic to the economy and therefore constant; the government controls the economy with the tranfer rate $\beta(t)$. The shock therapy consists in lowering abrubtly β from the high level of communism to the small level of capitalism. Gradualism implies a smoother

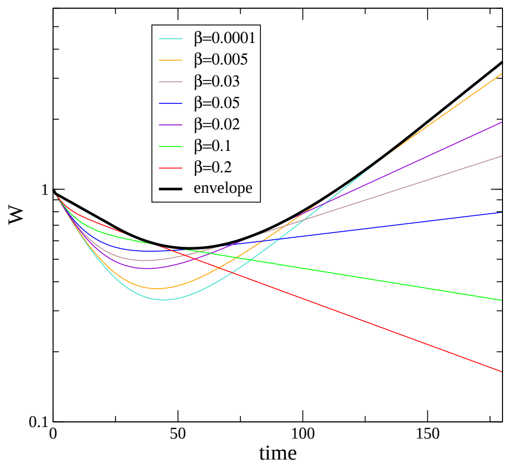

FIG. 2: Total economic output versus time for various values of the transfer rate β ($\alpha_1 = 0.02$, $\alpha_2 = -0.05$, $w_1(0) = 0.1$, $w_2(0) = 0.9$), and the upper envelope (

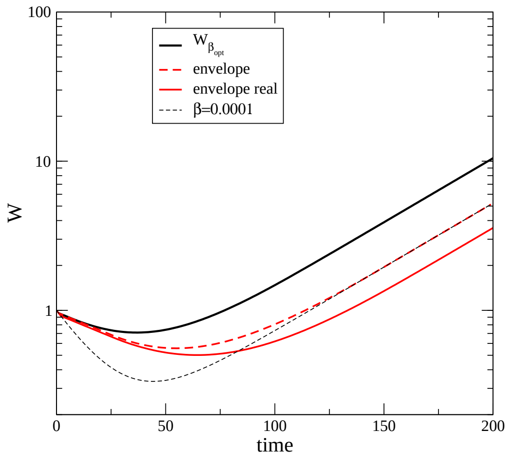

FIG. 3: Total economic output W for the optimal policy (black line), the naive envelope-based gradual policy (red lines) and the shock therapy (dashed black line). Same parameters as in Fig 2.

mathematical function for $\beta(t)$. Figure 2 plots various scenarios for $W(t)$ and shows the influence of β on the outcome. The cases with constant small β correspond to shock therapy. They are characterized by a deep recession and both a faster final growth rate and accordingly a higher GDP. Therefore, after many years, the tenants of this policy are vindicated since their courageous but harsh recommendations are proved correct as regards the growth rate of GDP and value *compared* to other static policies. This view is right, but only in a static context, as it maximises the *final* growth rate, not the instantaneous one, therefore not the actual GDP (see below), and deep recession ensues.

A. Naive dynamical policy: envelope

Few experiences are more frustrating for a politician than to have implemented a policy that will lead to the recovery of one's country, but too late from him to be re-elected. Instead of heroically jeopardizing one's political career, one should ask how to implement a policy that would avoid most troubles.

There is another way of looking at this figure: what if one could stay on the upper envelope of all the scenarios and thereby also maximising the GDP *and* the final growth rate? Running a thousand scenarios and selecting at each time t the value of β that gives the maximum yields Fig. 2: taxes should be kept maximal for a while, then β decreases exponentially fast in the region encompassing the worst phase of the recession, and then decreases faster than exponentially. Regretfully for a head of state, the minimum of the upper envelope is delayed compared to the outcomes of low-tax policies, but at much higher GDP. Even worse, in practice it is impossible to stay on the envelope

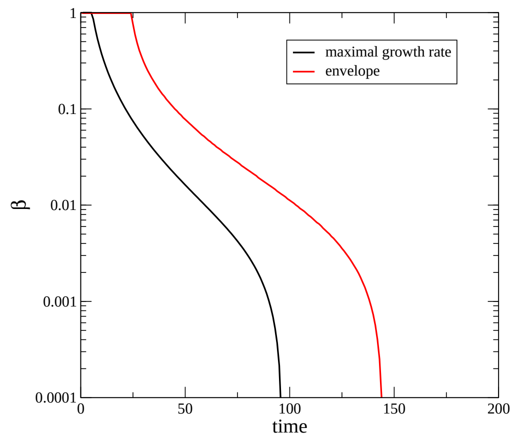

FIG. 4: Optimal value of the transfer rate β versus time for the naive envelope-based policy (red line) and the optimal policy (black line) (same parameters as in Fig. 2).

(see Fig. 3), because at time t, the actual values of w_1 and w_2 of $W(t)$ are not under direct control.

Curiously, $W(t)$ actually obtainable by using this method is also relatively well fitted with Eq. 1, i.e. with constant parameters, and gives for the curve reported in Fig. 2 $\lambda^+ \simeq 0.020$, $\lambda^- \simeq -0.019$, and $f = 0.068$ with uncertainties smaller than a percent, whereas the initial values were 0.02, -0.05 and 0.1, respectively. In other words, the effective shrinking rate is reduced, the rate of growth of the expanding sector remains unchanged (which is needed in order to retain the same final growth rate), while the apparent fraction of the expanding sector decreases much; interestingly, the difference between the fits of the envelope itself and the attainable $W(t)$ is limited to f: the envelope has the same apparent f as the individual runs.

B. Optimal policy: maximal W

Maximizing W reduces to the maximization of the growth rate with respect to β: $\frac{\partial \dot{W}}{\partial \beta} = 0$. This leads to a transcendental equation to be solved numerically at each time step. The resulting $W_{\beta_{\text{opt}}}$ is reported in Fig. (3), which shows unambiguously the benefits of the proposed optimal dynamical policy, that is, of gradualism with respect to shock therapy. Indeed, the value of the GDP in the recovery phase is increased several folds with respect to static policies and naive envelope-based dynamical policies, while sharing the same asymptotic growth rate. We therefore claim that shock therapies are inadapted to economies in crises as regards GDP. Patience and gradualism are better solutions in this kind of situations.

Looking at the optimal value of β (Fig. 4) reveals that indeed taxes should decreases rapidly, but not instantaneously. This means that the intuition behind shock therapies is correct, but only in the later stages of the time evolution. What matters is the road to minimum taxes, all the more since the economy follows multiplicative processes: optimizing it may change tremendously the fate of countries and people, as shown by the results of envelope-based and optimal policies.

Fitting $W_{\beta_{\text{opt}}}$ with Eq. (1) yields $f \simeq 0.80$, $\lambda^+ \simeq 0.20$ and $\lambda^- \simeq -0.027$. Therefore, the optimal policy both increases the apparent fraction of the growing part of the economy and decreases the apparent rate of shrinking of the decaying sector, while of course keeping constant the final rate of growth.

C. Detecting static, envelope-based, and optimal policies

A somewhat frustrating result of the previous two dynamical policies is the impossibility to distinguish them from a static one. Indeed, in the absence of additional information about the applied economic policies, one cannot reconstruct it from the GDP time series. For that purpose, one would need data about at least two sectors. Plotting $\Delta = w_1/w_2$ as function of time allows one to distinguish a static, envelope-based and optimal policy, as reported in Fig. 5: a static policy has a negative curvature, while dynamic ones start with a flat line, followed with a positive curvature and then an inflexion point. All of them reach the same asymptotic values since one imposed a minimum $\beta = 0.00001$ in order to compare the three policies. It may be difficult in practice to discriminate with naked eyes an

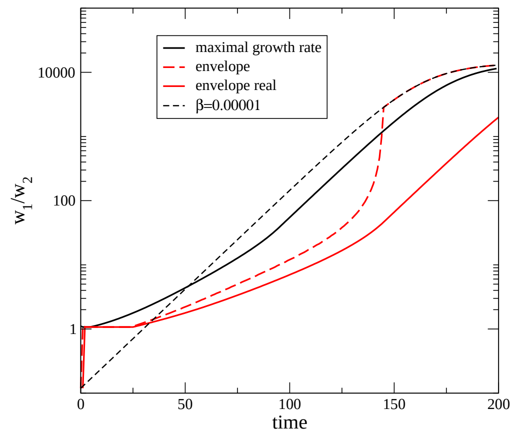

FIG. 5: Sector inequality as a function of time for the optimal policy (black line), the naive envelope-based gradual policy (red lines) and the shock therapy (dashed black line). Same parameters as in Fig 2.

envelope-based policy from the optimal one, but easy to detect a static policy[9]. However, in further investigations one could measure w_1 and w_2, and thus determine all the parameters of the model, including $\beta_{\mathrm{opt}}(t)$.

V. DISCUSSION

Economic transition from a fully centralized state to market economy is a complex process that left durable negative feelings to those who experienced it. How to deal with it is expectedly not consensual [2, 4], especially during times when economic criteria and institutions fail [5]. Our simplified model illustrates how much influence a redistribution policy has on the absolute value of the GDP.

Although it is common to separate recession and recovery into several phases (see e.g. [1, 2]), our model considers them as two aspects of the same global dynamical economic process. Our approach constrasts with the traditional factor-based analysis where one must separate the two phases of the dynamics in order to extract possibly relevant factors; it suggests rather that this type of regressions should be made on the fitting factors that we used rather than directly on the GDP. Even if taxation policies are by far not the only parameter at play, our simple model reproduces the typical patterns of recession and recovery, and strongly suggests that an optimal gradualistic approach both maximises the final growth rate, the actual GDP and minimizes the duration and severity of recession. Our model implies that one can predict during the recession the location of the minimum and the growth rate of the recovery phase provided as function of the tax policy.

Arguably, governments' ability to facilitate structural changes by economic activity transfer was much weakened more or less during the fall of the Iron Curtain. Accordingly, even if they had in mind the optimal policy (there is evidence that most of them did not), the result was beyond their crumbling control. However, one of the most important results of this paper is the existence of an optimal policy and the considerable speed at which the economic transfer rate should decrease; in practice however, there is a maximal speed of transition, due for instance to human workforce transfer between sectors [1, 5], which may prevent from applying the optimal policy. However, as regards the GDP, only the effective rate of transfer matters: should for instance a governement try to set a constant rate, but have an economic influence collapsing gracefully following the optimal path, it would have implemented unwillingly the optimal policy. This is to say that the timing of the policy has an enormous influence on the final GDP values: should the transition have been slightly less brutal in many if not all the considered countries, it would have been much closer to optimal. On the other hand, a too "gentle" transition policy might have lead to a worse current state. In short, the optimal policy is best described as gradual shock therapy.

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[6] Gur Yaari, Sorin Solomon, Kamil Rakocy, and Andrzej Nowak. Microscopic study reveals the singular origins of growth. ; lessons from the liberalization of the soviet bloc. *to appear in EPJB*, 2007.
[7] Although our discussion is limited to 2 sectors, it can be extended to N sectors, as in [6].
[8] In Ref. [1], a model that also produces Eq. (1) *in fine* is proposed.
[9] It is clear that increasing taxes will also lead to a negative curvature; however, we assume that the transition from communism to capitalism needs a decrease of taxes.

APPENDIX A: MODEL FITTING

In the following table, SRS stands for square residuals sum, SRM for square residuals mean, RSRS for relative square residuals sum, RSRM for relative square residuals mean; years indicates the number of years along which the fit has been done. The table contains a wealth of data which can be further exploited and whose potential has been only scratched by the present publication.

Country	f	α_1	$1-f$	α_2	SRS	SRM	RSRS	RSRM	years
Albania	59.5	1.064	40.4	0.367	301.97	17.763	402.92	23.701	17
Armenia	14.9	1.159	85.0	0.722	672.87	39.581	1639.2	96.427	17
Azerbaijan	20.1	1.120	79.8	0.639	37.121	2.6515	121.09	8.6493	14
Bahamas	70.8	1.061	29.1	0.662	4.6524	0.4652	4.4493	0.4449	10
Belarus	38.0	1.089	61.9	0.737	151.97	9.4982	241.12	15.070	16
Bolivia	61.9	1.049	38.0	0.775	16.529	0.9183	18.257	1.0143	18
Chile	51.2	1.079	48.7	0.522	56.499	3.3235	37.112	2.1830	17
Cuba	45.7	1.061	54.2	0.690	197.90	11.641	248.80	14.635	17
Estonia	40.7	1.083	59.2	0.688	148.97	8.7630	211.69	12.452	17
Finland	82.1	1.036	17.8	0.367	79.249	4.6617	65.940	3.8788	17
Georgia	17.0	1.079	82.9	0.573	279.77	16.457	1306.0	76.827	17
Hungary	64.0	1.046	35.9	0.694	38.588	2.1438	45.955	2.5530	18
Kazakhstan	15.6	1.137	84.3	0.837	105.25	6.1914	187.13	11.007	17
Latvia	27.4	1.087	72.5	0.651	245.92	14.466	508.97	29.939	17
Lithuania	31.1	1.082	68.8	0.723	214.11	12.594	411.24	24.191	17
Mongolia	64.5	1.045	35.4	0.615	156.65	9.7910	143.21	8.9512	16
Poland	91.3	1.042	8.7	0.018	216.09	12.711	140.94	8.2906	17
Republic of Moldova	14.6	1.081	85.3	0.734	103.41	6.0831	395.33	23.255	17
Russian Federation	20.4	1.100	79.5	0.825	102.63	6.0373	181.19	10.658	17
Slovakia	72.9	1.042	27.0	0.367	102.28	6.0170	95.323	5.6072	17
Slovenia	79.5	1.040	20.4	0.367	29.452	1.7324	27.235	1.6020	17
Sweden	88.1	1.029	11.8	0.549	28.900	1.7000	23.565	1.3861	17
Turkmenistan	29.9	1.080	70.0	0.809	291.63	17.154	538.89	31.699	17
Ukraine	6.71	1.153	93.2	0.835	241.22	14.189	656.64	38.626	17
Uzbekistan	42.6	1.074	57.3	0.829	77.763	4.5743	87.654	5.1561	17
Romania 1	60.4	1.075	39.5	0.526	6.2065	0.8866	8.4506	1.2072	7
Romania 2	60.3	1.070	39.6	0.656	9.3063	0.8460	9.1387	0.8307	11
Czech Republic 1	75.0	1.047	24.9	0.422	3.3792	0.4827	4.2602	0.6086	7
Czech Republic 2	38.1	1.102	61.8	0.927	5.8393	0.5308	5.3579	0.4870	11
Bulgaria 1	0.2	1.967	99.8	0.929	22.261	2.7827	30.514	3.8143	8
Bulgaria 2	64.5	1.051	35.4	0.463	5.0247	0.4567	6.0662	0.5514	11